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FORCED CONVECTION HEAT TRANSFER OVER A SEMI-INFINITE PLATE

J. Dey
Mechanical Engineering Department

and

G. Nath
Applied Mathematics Department, Indian Institute of Science,
Bangalore 560012, India

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NOMENCLATURE

A, B,	constants;
f, f_1,	zeroth and first order functions, equation (7);
Pr,	Prandtl number;
R_e,	Reynolds number;
T,	dimensionless temperature;
t,	temperature;
u, v,	velocity components in x, y-directions;
U,	free-stream velocity;
x, y,	Cartesian co-ordinates.

Greek symbols

σ, η,	parabolic co-ordinates;
θ, θ_1,	zeroth and first order temperature functions, equation (8);
ν,	kinematic viscosity;
φ,	non-dimensional stream-function.

Subscripts

w,	denotes conditions at the wall;
∞,	denotes conditions in the free-stream;
0,	denotes local value of σ at the wall;
$\sigma,\eta,\sigma\sigma,\eta\eta$,	denotes partial derivatives with respect to σ, η, $\sigma\sigma$, and $\eta\eta$, respectively.

1. INTRODUCTION

A very interesting heat transfer situation occurs for flow over a semi-infinite flat plate, which is maintained at a temperature that is inversely proportional to the square root of the distance (x) from the leading edge ($x = y = 0$). In such situation, the local heat transfer coefficient is found to be zero [1, 2] for $x > 0$ though the wall temperature (t_w) is different from the free-stream temperature (t_∞). The fluid temperature changes during the flow process though there is no local heat transfer at the plate surface for $x > 0$. However, these analyses [1, 2] were based on the boundary-layer approximation, which is valid in the far downstream direction. As such, the heat transfer process needs to be re-examined for small x defining upstream flow.

The aim of this paper is to report on the role of plate length (x) in the heat transfer process for the flow geometry mentioned above. Parabolic co-ordinates, which are optimal for a flat plate [3], have been used to study the forced convection flow using Navier–Stokes equations where as in ref. [2] the combined effect of both forced and free convection was studied under the boundary-layer approximation.

2. GOVERNING EQUATIONS

Let u and v denote the two velocity components in the x and y directions, respectively. The local temperature is denoted by t and the uniform free-stream velocity parallel to x is denoted by U. The non-dimensional parabolic co-ordinates (σ, η) are defined such that

$$x + iy = \gamma(\sigma + i\eta)^2/2U \tag{1}$$

$$x = \gamma(\sigma^2 - \eta^2)/2U$$

$$y = \gamma\sigma\eta/U \tag{2}$$

where γ is the kinematic viscosity, η is retained as positive so that positive σ represents the positive y-space of interest to us.

For steady, incompressible, laminar flow over a semi-infinite flat plate the 2-dim. vorticity equation and the energy equation in parabolic co-ordinates are

$$P_{\sigma\sigma} + P_{\eta\eta} + \varphi_\sigma P_\eta - \varphi_\eta P_\sigma = 0, \tag{3}$$

$$t_{\sigma\sigma} + t_{\eta\eta} + Pr[\varphi_\sigma t_\eta - \varphi_\eta t_\sigma] = 0. \tag{4}$$

Here suffixes σ and η denote derivative with respect to σ and η, respectively. P is given by $P = (\varphi_{\sigma\sigma} + \varphi_{\eta\eta})/(\sigma^2 + \eta^2)$ and φ is the non-dimensional stream function. Pr denotes the Prandtl number. In this analysis it is assumed that $t_w \propto x^{-1/2}$, i.e. the plate temperature (t_w) is inversely proportional to the square root of the distance from the leading edge. The non-dimensional temperature $T(\sigma, \eta)$ is defined by $T = (t - t_\infty)/(t_w - t_\infty) = (t - t_\infty)/Ax^{-1/2}$, where A is constant. Since the wall is defined by $\eta = 0$, we have $(t - t_\infty) = BT/\sigma$, where B is another constant. The energy equation (4) becomes

$$(T/\sigma)_{\sigma\sigma} + (T/\sigma)_{\eta\eta} + Pr[\varphi_\sigma(T/\sigma)_\eta - \varphi_\eta(T/\sigma)_\sigma] = 0. \tag{5}$$

The boundary conditions are

$$\varphi(0) = \varphi_\eta(0) = 0;\ T(0) = 1;$$

$$\varphi(\infty) \simeq \sigma\,\eta;\quad T(\infty) \to 0. \tag{6}$$

The dimensionless stream function (φ) and the temperature (T) are expanded [4, 5] about $\sigma_0 = (2R_e)^{1/2}$, where R_e ($= Ux/\gamma$) is the local Reynolds number, on the plate surface as

$$\varphi = \sigma[f(\eta) + (\sigma - \sigma_0)f_1(\eta) \ldots\ldots] \tag{7}$$

$$T = [\theta(\eta) + (\sigma - \sigma_0)\theta_1(\eta) \ldots\ldots] \tag{8}$$

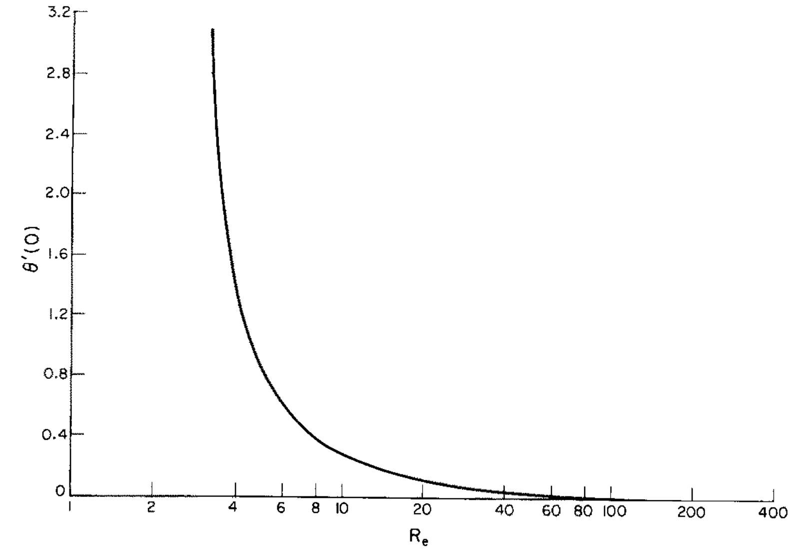

FIG. 1. Effect of local Reynolds number on the non-dimensional wall temperature gradient.

which are dictated by the boundary conditions. Substituting equations (7 and 8) in equations (3–5) and retaining the first term only, we obtain

$$f'''' + f'''\left[f - \left(\frac{4\eta}{\sigma_0^2 + \eta^2}\right)\right] + \left(\frac{f''}{\sigma_0^2 + \eta^2}\right) \times [(\sigma_0^2 - \eta^2)f' - 2\eta f] = 0, \quad (9)$$

$$\theta'' + [2\theta/\sigma_0^2] + Pr[f\theta' + f'\theta] = 0. \quad (10)$$

The corresponding boundary conditions are

$$f(0) = f'(0); \quad \theta(0) = 1;$$

$$f(\infty) = \eta; \quad f'(\infty) \to 1; \; \theta(\infty) \to 0. \quad (11)$$

Equations (9 and 10) reduce to the boundary-layer equations of ref. [2] (differing by a factor of 2 in the highest order derivative due to scaling) as $\sigma_0 \to \infty$. It may be noted that leading edge ($\sigma_0 = 0$) situation can be described by the vorticity equation (9) but the energy equation (10) reduces to a singular perturbation problem as $\sigma_0 \to 0$ [6]. Hence, we have considered $\sigma_0 > 0$. Equation (10) after integration yields

$$\theta'(0) = \int_0^\infty (2\theta/\sigma_0)\mathrm{d}\eta. \quad (12)$$

Here it is assumed that $\theta'(\infty) = 0$, which is, of course, obvious. Equation (12) shows that the local heat transfer coefficient is not independent of the plate length (for given U and γ) and becomes independent of the plate length as $\sigma_0 \to \infty$, representing the result of ref. [1] and the forced convection result of ref. [2]. In other words, in the far downstream region the local heat transfer coefficient is zero. Also, equation (12) shows that in the upstream region (for low and moderately large Reynolds numbers) heat is transferred from the fluid to the wall and not from the wall to the fluid.

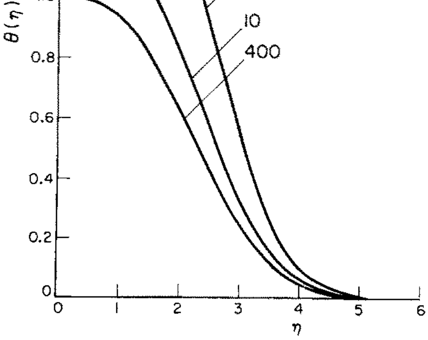

FIG. 2. Non-dimensional temperature profiles for various values of the local Reynolds number (R_e).

3. RESULTS AND DISCUSSION

Equations (9)–(11) were solved numerically for $Pr = 0.7$ and for different Reynolds numbers. Results, shown in Figs. 1 and 2, are approximate due to series truncation. Results of Davis [4] show that higher truncation will increase the accuracy. The positive local heat transfer coefficient [$\theta'(0)$] profile (Fig. 1) approaches an asymptotic value at $R_e \simeq 3.2$. As such, the solution of equation (11) for $R_e < 3.2$ needs separate investigation, which is being carried out. $\theta'(0)$ approaches the value obtained under the boundary-layer approximation as $\sigma_0 \to \infty$, i.e. $\theta'(0) \to 0$ as $R_e \to \infty$. Temperature profiles (Fig. 2) exhibit large overshoots for low Reynolds numbers and approach the profile shape for which $\theta'(0) = 0$ with increasing Reynolds number ($R_e = 400$). The entire heat transfer to the fluid occurs at the singular point ($x = 0$) and then redistribution takes place during the flow development over the plate. During this process, the wall is active. After the redistribution is complete, the temperature distribution becomes that of ref. [1] or ref. [2] for which the wall is inactive.

4. CONCLUSIONS

From the above discussion we can conclude that the observation [1, 2] that the local heat transfer coefficient is independent of the plate length is true in the far downstream region. In the upstream region ($R_e \geqslant 3.2$) the local heat transfer coefficient depends on the plate length.

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